How early is early enough?
Solving the place-value problem in first grade

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ABSTRACT

Place-value is a problem for American, but not for Asian children whose counts make the base-10 patterning of the number system explicit. A pilot curriculum using an explicit base-10 count and a single manipulative was initially tested with kindergarteners. Results showed the children mastering place-value, and using that knowledge to solve single- and double-digit addition and subtraction problems. Given that later achievement in math is highly correlated with early achievement, the current study asked is first grade early enough? These results show that, despite having already learned the English language count, first grade is early enough to master an explicit base-10 count and solve the place value problem.

Key words: place-value, early childhood education

BACKGROUND: STRUCTURING THE SOLUTION

My expertise is not in mathematics, but in problem-solving per se. My solution to the place-value problem was structured in what Newell and Simon (1972) called a problem space. A problem space has three parts: an initial state, a goal state, and between the two, a search space in which a solution path is constructed. Constraint pairs are tools for structuring a solution path (Reitman, 1969; Simon, 1973; Stokes, 2006, 2010, 2013). One of each pair precludes something specific in the initial state, the other directs or promotes search for a substitute.

The Place-Value Problem

Place-value is problematic for American, but not for Asian children, whose languages clarify the recursive patterning of the base-10 count (Fuson, 1990; Miura & Okamoto, 2003). Table 1 shows an abbreviated version of the Asian (Korean, Chinese, Japanese) count from 1 through 29.

Table 1: Explicit base-10 count.

	Ones		Tens		Twenties
		10	*ten*	20	two-*ten*
1	one	11	*ten*-one	21	two-*ten*-one
2	two	12	*ten*-two	22	two-*ten*-two
3	three	13	*ten*-three	23	two-*ten*-three
4	four	14	*ten*-four	24	two-*ten*-four
5	five	15	*ten*-five	25	two-*ten*-five

6	six	16	*ten*-six	26	two-*ten*-six
7	seven	17	*ten*-seven	27	two-*ten*-seven
8	eight	18	*ten*-eight	28	two-*ten*-eight
9	nine	19	*ten*-nine	29	two-*ten*-nine

There are several important things to notice about the count. First, there are only ten number names (1 to 10), which combine to form higher numbers. Second, ten appears in every number above ten: 11 is *ten*-one; 21 is two-*ten*-one. In contrast to American children who think of numbers as concatenations of ones (21 means 21 ones), Asian children think of numbers as tens and ones (21 means 2 tens and 1 one). Thinking this way, place-value is not a problem.

The Solution

The solution was not simply to introduce an explicit base-10 count, but to embed it in a curriculum that taught children to think mathematically, in large meaningful patterns. Table 2 shows the problem space. The initial state was current curricula. The goal state was the new curriculum. Its criterion was "thinking in numbers, symbols, and patterns."

The first constraint pair precluded the English language count and promoted an explicit base-10 count. The second and third were selected to further satisfy the new criterion. By "words" I meant videos with cartoon characters, as well as work sheets with stories and word problems that can distract children from the strictly numeric. The single manipulative was meant, like the abacus, to make base-10 numbers and patterns tangible and concrete.

Table 2*:* Problem Space for New Math Curriculum

Problem Parts	Description		
Initial State	Current curricula		
Search Space	Constraint pairs		
	Preclude		*Promote*
	English language count	→	Explicit base-10 count
	Words	→	Numbers, symbols, patterns
	Multiple manipulatives	→	Single manipulative
Goal State	New curriculum		
Criterion	Thinking in numbers, symbols, and patterns		

Figure 1 shows the manipulative, called the count-and-combine chart, with the numbers 1 through 5. The movable parts, meant to be combined and recombined in multiple ways are number names, numbers, symbols, and colored boxes (which the children call "blocks") representing ones. Children begin by reciting the rows. The top row is read "number one same as word one equals one block." Figure 2 shows the chart with the numbers 10 to 15 (ten-five). Notice that ten is represented as a unit, by a single block simply marked "10." Notice too, the similarities between the charts. In each, the block pattern mirrors the regularities in the count: three equals 3 one blocks; ten-three equals one 1 ten block and 3 one blocks.

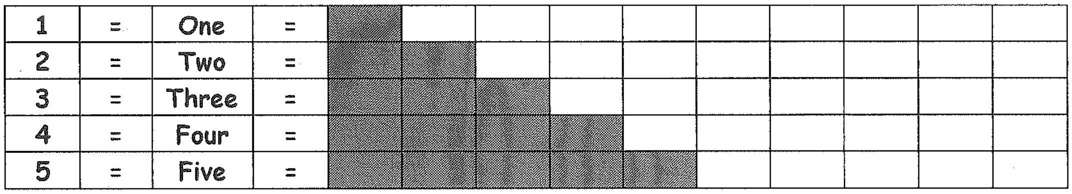

Figure 1. Count-and-combine chart for numbers 1 to 5.

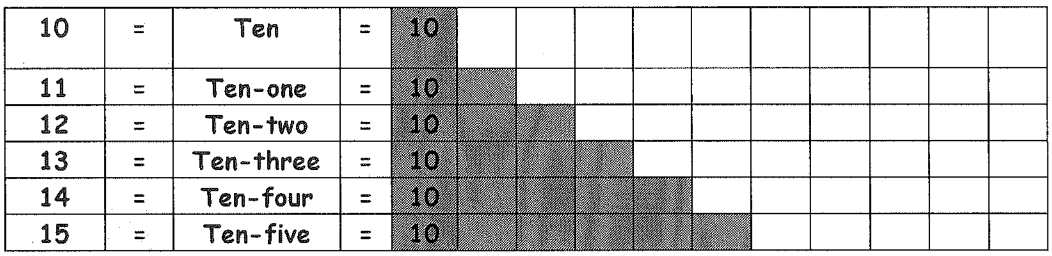

Figure 3: Count-and-combine chart for numbers 10 to 15

IMPLEMENTING THE SOLUTION IN FIRST GRADE

To see if first grade was early enough to successfully introduce an explicit base-10 curriculum, a design similar to the kindergarten study was implemented. One first grade class (the test group) used the new curriculum; another class (the comparison group) in the same school used the district-wide curriculum.

METHOD

Participants

Participants were twenty eight students from two first grade classes at a suburban public school. The classes were sorted by gender (to balance the number of boys and girls) and not by ability. One class served as the test, the other as the comparison, group. Both adhered to the New Jersey State Standards in math. The pilot group, like the comparison, used materials from Scott-Foresman/Addison-Wesley (*enVisionMATH*, 2011) for topics like patterns, graphs, and measurements. The new curriculum (*Only the NUMBERS Count*©) replaced materials covering numbers and numeric relations. The time for math was allotted equally across groups.

Test Group

Students. At the start of the school year, 21 students (11 female, 10 male) participated in pre-testing. Six of these (3 female, 3 male) were not post-tested, and are not included in the analyses. Of the remaining 15 students, 8 were female and 7 male; 1 was Asian, 1 was African-American, 2 were multiracial, 6 were Hispanic and 5 were White. Six were economically disadvantaged (i.e., qualified for free lunch). All were proficient in English, of these, 12 were bi-lingual, speaking a language other than English at home. Mean age at the time of pre-testing was 80 months, range was 72 to 94 months.

Teacher. The teacher had participated in the development of the new curriculum two years earlier while teaching and introducing the curriculum to her kindergarten class. She had previously used both *Everyday Math* and *Scott Foresmann/Addison Wesley* (Math Series Copyright 2008). At the start of the school year, she had five years teaching experience, four teaching kindergarten and one (the previous year) teaching first grade.

Comparison Group

Students. At the start of the school year, 18 students (11 female, 7 male) participated in pre-testing. Five of these (3 male, 2 female) were not post-tested, and are not included in either the analyses. Of the remaining 13 students, 9 were female and 4 male; 8 were Hispanic and 5 were White. Ten were economically disadvantaged (i.e, qualified for free lunch). All were proficient in English, of these, 8 were bi-lingual, speaking a language other than English at home. Mean age at time of pre-testing was 78 months, range was 71 to 81 months.

Teacher. The teacher for the comparison group followed the district's current curriculum (*enVisionMATH*). She had previously used the same earlier Scott Foresman curriculum as the test teacher. She had three years teaching experience, all in first grade.

Materials and Procedure

The study was conducted in three phases. Phase 1 involved pre-testing to assess children's prior knowledge. Phase 2 involved observing test and comparison classrooms on a weekly basis. Phase 3 involved post-testing to assess what had been learned. Testing and observations were conducted by the primary experimenter and three undergraduate research assistants (all female).

Phase 1: Assessing Prior Knowledge

There were two reasons for pre-testing at the start of the school year. The first was to see if the classes were equivalent in mathematical ability and/or preparation. The second was to compare specific changes in numeric understanding at post-test. Table 3 summarizes items on the pre-test. The place value task was a variant of the Choose the Larger Number Test, which requires picking the larger of a pair of numbers (Fuson & Briars, 1990).

Table 3. Pre-Test Tasks

Category	Content
Count to 100	By ones. Counting was coded as correct up to the first error (If a child counted 11, 12, 15, her score would be 12, the highest number correctly counted). By tens. Again, counting was coded as correct to the first error.
Number of 10s	Children were asked how many tens there are in 30 and in 50.
Number/symbol identification	Children read aloud numbers (1, 2, 3, 4, 5, 7, 8, 12, 15, 17, 20, 32) alone or combined with symbols (plus, minus, equals) in problem formats like 2 + 2 = 4.
Place value	Children were asked (a) to read aloud the written numbers 16, 25, 31, 56, 11; (b) to tell the experimenter which of each pair was bigger and (c) to explain their answers.

Addition	Children solved four problems in which one or both addends were less than 10 (3 + 5, 6 + 6, 9 + 7, 10 + 8). They were asked to (a) read each problem, (b) solve it, and (c) explain what they did.
Subtraction	Children solved three problems in at least one number was a single digit (5 – 3, 10 – 5, 13 – 6), and one in which both numbers were identical (10 – 10). They were asked to (a) read the problem, (b) solve it, and (c) explain what they did.
Combinations	Children make up two different addition problems for each of the following sums: 8, 10, 16, 22. The problems looked like this: __ + ___ = 8.
Word Problems	Children solved two addition and three subtraction word problems.

Phase 2: Observing the two classes

Here I illustrate (in simplified form) how double-digit addition was introduced in February. I chose these lessons because both classes used manipulatives to represent 10. I also point out how one clarified place-value, and how the other obscured it.

Test class

Children were given baskets with blocks representing ones, tens, plus and equals signs. The teacher wrote the addition problem on the board. The children used the 10 and 1 blocks to make the addition combination. They first added the tens and then the ones. The count makes this separation obvious. It also makes place-value obvious. The top row of Figure 4 shows the problem 21 + 22. It was read "two-ten-one plus two-ten-two." The children added the ten blocks first: two-ten plus two-ten is four-ten. Then they would add the ones: one plus two is three. The answer, four-ten-three, would also be written in numbers and symbols.

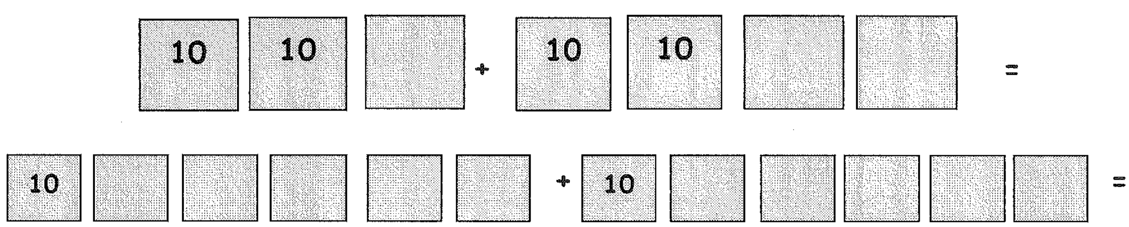

Figure 4. Top row ten and one blocks combined for addition problem 21 + 22. Bottom row: ten and one blocks for problem 15 + 15.

The second problem (15 + 15) shows what happens if the ones add up to 10 or more. The problem would be read as "ten-five plus ten-five." The children would combine the blocks to look like the bottom row of Figure 4. First they would add the tens: ten plus ten is two-ten. Then they would add the fives: two 5s are 10. Finally, they would add this ten to the two-ten. The answer, three-ten, would also be written in numbers and symbols.

Notice that place-value is clear in the blocks, in the adding, and in the answer.

Comparison Class

In a lesson on skip-counting by 10s, children were given work sheets, cube trains for 10s and individual cubes for 1s. Problems on the work sheet included illustrations of cube trains. If the problem on the work sheet were 28 + 30 = , it would look like Figure 5. The children would skip count from 28: (once) 38 (twice) 48 (three times) 58. The answer, 58, would be filled in on the work sheet.

If the problem were 51 + 40, the children would skip count four groups of 10. Starting with 51, they would skip once to 61; skip twice to 71; skip three times to 81; and finally, skip four times to 91.

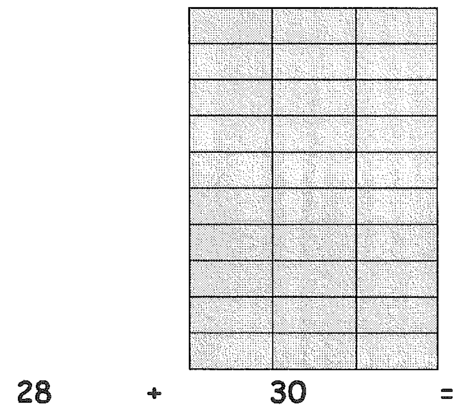

Figure 5. Three cube trains used to represent 30.

Shouldn't a lesson like this one, concentrating on 10s, contribute to understanding place value? I don't think so, and for two reasons. First, a cube train (or stick) is both seen and called a group of 10 ones. Counting by tens means counting by *groups of ten ones.* This kind of bundled manipulative contributes to American children thinking of numbers as concatenations of ones. There is clear evidence for this. When asked to represent a number (42) using big blocks that new items. Table 4 describes tasks on the post-test.

represented 10s and small ones that represented ones, Asian children gathered 4 American children counted out 42 one blocks (Miura & Okamoto, 2003; Miura et al.,1988). Second, our number names nullify the counting by 10s: 91 is called ninety-one. When the 10s place is not named, the tens lose their place-value.

Phase 3: Assessing New Knowledge

The post-test was administered at the close of the school year.

Table 4. Post-Test Tasks

Category	Content
Count to 100	Identical to pre-test.
Number/symbol identification	Identical to pre-test.
Number of 10s	Identical to pre-test.

Place value	Identical to pre-test.
Addition	Children solved three problems from the pre-test (3 + 5, 6 + 6, 9 + 7) and three new problems in which both addends were 10s (10 + 18, 21 + 11, 35 + 17).
Subtraction	Children solved three problems from the pre-test (5 – 3, 10 -5, 13 – 6) and three new problems (20 – 20, 32 – 15, 25 – 19).
Combinations	Children were asked to make up two problems with two addends (__ + __ = 8) and one problem with three (__ + __ + __ = 8) for each of the following sums: 8, 10, 16, 25.
Word Problems	Children solved four addition and two subtraction word problems. One was taken from the pre-test; two were more difficult variants of the original problems; the others were new.

RESULTS

Average accuracies, calculated as percentage correct, are shown in Table 5. Items in bold-face are those in which the test group performed noticeably better than the comparison group.

Both groups improved and, at post-testing, were comparable in counting, number/symbol identification, and single-digit addition. However, only the test group was proficient at place-value. Counting correctly is sufficient to know the ordering, but not the place-value of numbers in a count. On all other items, the test group had noticeably higher average accuracies at post-testing than the comparison group.

Table 5. Pre-and post-test scores for first grade classes: Average accuracies.

	Group							
	Test				Comparison			
	Pre		Post		Pre		Post	
Measure	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Count to 100	80.73	25.57	96.06	15.23	88.84	27.02	96.76	11.36
by 10s	74.00	44.68	100.00	.00	65.38	43.51	100.00	.00
Tens in 30 & 50	40.00	50.70	100.00	.00	23.07	43.85	84.61	37.55
Number-Symbol Identification	95.13	9.64	100.00	.00	94.46	7.75	100.00	.00
Place Value	58.33	46.92	**93.33**	25.81	.00	.00	**.00**	.00
Same*	66.67	48.79	**93.33**	25.81	.00	.00	**.00**	.00
Addition								
Single	53.33	29.65	91.13	19.79	73.69	28.67	84.69	21.98
Double			**71.20**	35:34			**20.53**	34.86

Subtraction								
Single	39.80	40.15	**91.20**	15.10	48.38	37.40	**72.00**	26.64
From same	60.00	50.70	**100.00**	.00	38.46	50.63	**76.92**	48.85
Double			**36.66**	39.94			**.00**	.00
Addition Combos								
Doubles	64.20	34.32	**92.53**	13.11	63.69	32.75	**54.07**	34.01
Triples			**90.00**	20.70			**32.69**	40.03
Word Problems								
Addition	93.33	17.59	**91.66**	20.41	80.76	25.31	**61.53**	21.92
Subtraction	30.00	36.83	**76.66**	41.69	42.30	34.43	**34.61**	42.74

*Place Value Same: Test item with identical numeral in tens and ones places – number was 11.

ASSESSING THE SOLUTION

Why did the new curriculum work so well? To answer this question, I review the contributions of the explicit base-10 count and the single manipulative.

Contribution of the Count: Thinking in Base-10

Asian children have long out performed American not only on place-value, but also on multi-digit addition and subtraction (Fuson & Kwon, 1990; Song & Ginsberg, 1987; Stiger, Lee, & Stevenson, 1990). In like manner, the test class – using an English language version of the Asian count – out performed the comparison class on multi-digit addition and subtraction.

I attribute their performance, in large part, to *thinking* in base-10. Think of it this way. The explicit base-10 count named the place of each number. Children using the count *thought* of 24 as 2 tens and 2 fours. *Thinking* in this way, the multi-digit problem 24 + 37 became "two-ten-four" plus "three-ten-seven." As shown in the sample lesson, the children would first add the tens (to get five-ten), and then add the ones (to get ten-one). Again adding the tens (five-ten plus ten) they would get six-ten, which appended to the one gives the answer, six-ten-one. *Thinking* in base-10 facilitated multi-digit calculation.

Contribution of the Count-and-Combine Chart: Practicing in Base-10

Experts solve problems using large meaningful patterns in their areas of expertise (Ericson, 2006; Newell & Simon, 1972). The count-and-combine chart was designed to make numbers and symbols real, concrete, things with highly visible, patterned relationships among them. The similarities between the charts for lower and higher numbers (1-10, 10-20) visualized and emphasized the reiterative pattern of the number system. Importantly, the moveable "blocks" stood for numbers and symbols and nothing else. Equally importantly, the "ten" block stood for 10 and nothing else.

Practice with the chart and the moveable blocks was iterative and elaborative. Children practiced the *pattern of the base-10 count* by reciting and reconstructing each chart. They practiced the *patterns of base-10 solutions* for addition and subtraction problems. On a daily basis, they recomposed and decomposed combinations of tens and ones blocks to solve problems. As the numbers increased, so did the possible combinations and the complexity of the problems. This kind of focused, continuous, incremental practice is similar to a Japanese first grade curriculum described by Murata and Fuson (2006) as "coherent, with fewer topics that build over a year," opposed to the typical "mile wide inch deep US pattern" (p. 454).

As I argued in an earlier section, practice using the ten block, which stood solely for 10 and not for a grouping of ten 1s, was critical to mastering both place-value and multi-digit computation. Using the ten block made practice in base-10 possible.

Answers to Anticipated Questions

Did children also use the regular count?

First graders in the test group, like kindergarteners in the earlier study (Stokes, 2013) were fluent in both the explicit base-10 and the regular count. They could interchangeably refer to the number 20 as "two-ten" or "twenty."

How did they do so well on word problems?

Word problems were not excluded from the curriculum, but they were only introduced after children acquired mathematical models on which to map them. Once children knew how to add and subtract, the teacher simply told them to think of things in the word problem as "blocks." As post-test results showed, the mapping helped solve the problems.

How were non-numeric items taught?

Materials from the district-wide curriculum were used for common core items like measurement, graphs, and shapes. Other items not included the core were also taught using the new count. For example, the count makes understanding money relatively simple. 100 is ten-ten; there are ten dimes in a dollar. Half of ten-ten is five-ten; there are two fifty cent pieces in a dollar, etc.

Were post-test differences due to the teachers?

This is impossible to assess since they were using different curricula. However, we can compare two kindergarten teachers who used the new curriculum for the first time last year. Data from district wide computerized testing (*Renaissance* STARMath) took place during the sixth month of the school year. In both classes, 93% of the children scored above grade level. Scores ranged from 0.5 (fifth month, kindergarten) to 2.3 or 2.4 (third or fourth month, second grade). Same curriculum, different teachers, comparable results.

How did the classes compare on the district wide tests?

Again, the test group scored higher than the comparison. Scores for children whose data was included in the present study[1] are reported here. Children were tested at the end of the school year (June).

In the test group, 71% of the children scored above grade level. Scores ranged from 2.0 (start of second grade) to 3.6 (sixth month, third grade). 29% scored below grade level. These scores ranged from was 1.5 (fifth month, first grade) to 1.6 (sixth month, first grade).

In the comparison group, 30% of the children scored above grade level. Their range was 2.1 (first month, second grade) to 2.7 (seventh month, second grade). 69% scored below. The range here was from 1.3 (third month, first grade) to 1.6 (sixth month, first grade).

Did the pilot curriculum have any lasting effects on mathematical performance?

To answer this question, recent district-wide computerized test scores (third grade, fifth month) for students from the original test and comparison groups were examined. The groups did not remain intact. By third grade, only 60% of the test group, and 72% of the comparison group were still in the school, and were distributed in three different classes.

Mean and median scores for the test group were both 4.3 (third month, fourth grade); scores ranged from 3.4 (fourth month,

[1] Only those who were both pre- and post-tested.

third grade) to 5.3 (third month, fifth grade). Mean and median scores for the comparison group were both 3.9 (ninth month, third grade); scores ranged from 3.2 (second month, third grade) to 4.5 (fifth month, fourth grade).

Overall, the performance gap was four months. A bigger difference appeared at the top: 44% of the test group scored at or above 4.5 (fourth grade, fifth month – a year ahead); only 8% of the comparison group did as well.

Immersion in Base-10: Implications ...

Immersion is critical to the new curriculum. Immersed in an explicit base-10 curriculum for an entire school year, first graders - like kindergarteners in an earlier study (Stokes, 2013) - had little difficulty in mastering place-value or double-digit addition and subtraction. The results of the two studies have two implications.

One, immersion can help kindergarteners and first graders meet common core requirements (1 and 2 OA: Operations and Algebraic Thinking, and 1 and 2 NBT: Numbers and Operations in Base Ten) for both first and second grades.

Second, immersion is inexpensive, intuitive, and enjoyable. The count-and-combine charts, along with the baskets of blocks, were all made by the teacher with foam core, velcro, and poster board. In comparison to the curricula she had previously used (*Everyday Math*, *Scott Forseman/Addison Wesley Math Series*), the test group teacher reported that the new curriculum was intuitive and easier – both for her to teach and for her students to learn.

... and Conclusion.

Early mathematical performance is predictive of later performance (Duncan, Dowsett, Claessens, Magnuson, Huston, & Klebanov, 2007; Stevenson & Newman, 1986). The current study asked if first grade was early enough to immerse children in an explicit base-10 count and curriculum. The answer was yes. The hope is that *solving the place-value problem early* will improve mathematical understanding and accomplishment later.

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