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Sources of Variability in Children's Problem Solving

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Variability is defined as the number of different ways in which something is done. Adaptive variability reflects varying in correct ways; error variability, in incorrect ways. This study examined two sources of variability in children's problem solving: age and reinforcement. First, third, and fifth graders at a suburban, public elementary school played a computer maze game with increasing variability requirements. Baseline variability levels were higher in fifth- than in first-graders. These differences disappeared when variability was required. Error scores increased with variability requirements, but were higher in first- than in third- or fifth-graders. Older students used more efficient strategies than younger ones. Students with high variability and low error scores shared response patterns and strategies, regardless of grade. Pedagogical implications of age and reinforcement effects are discussed.

INTRODUCTION

Variability has been conceived of as a continuum, with high and low levels at its extremes. Expected, reliable, repetitive behaviors lie closer to the low end, while surprising, novel behaviors lie closer to the high (Stokes, 1999, 2001). The words "repetitive" and "novel" indicate what variability refers to (how differently something is done) and the simplest way to measure it (counting the number of different responses/strategies used by an individual acquiring a skill, performing a task, or solving a problem).

Some children are consistently more variable in problem solving than others. In mastering single digit arithmetic problems, one uses three different strategies; another switches between five. For example, three possible strategies for adding 2 + 4 are "guess," "retrieve," or "sum." The last involves counting from 1 to 2 on one hand, from 1 to 4 on the other, and then counting up all the digits/fingers (Siegler & Jenkins, 1989). In learning to spell, individual children employ between two and six different strategies. These include "retrieve", "sound out," and the combination "retrieve/sound out" in which the child recalls one or more syllables and sounds out the others (Rittle-Johnson & Siegler, 1999).

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Gifted children are more variable than their age-mates in the area of their gifts (Stokes, 2001, 2005; Winner, 1996). For example, in a longitudinal study of drawing less talented children drew simple canonical representations (frontal views with a minimum of "necessary" features associated with an object). In comparison, the more talented added specific details, textures and shading to objects which, with experience, were rotated and foreshortened (Milbraith, 1998).

Suggested sources of such variability differences include the child's age (Sielger, 1994) and brain development (Bjorkland & Green, 1992), as well as early reinforcement for high or low variability (Stokes, 1999; Stokes & Balsam, 2001). In this paper, we examine a combined possibility: the child's age when first exposed to a domain in which high variability is rewarded. We begin with a brief overview of the benefits and the bases of high variability.

Benefits of High Variability

High Variability Accompanies Learning

Classic developmental theories (Piaget, 1955; Vygotsky, 1987) suggest that learning occurs during periods of increased variability. Current studies show just such temporary increases in variability when children expand their cognitive skills (Fujimara, 2001; Miller & Aloise-Young, 1995; Schauble, 1996) or their spatial/motor skills (Adolph, 1997; Bertenthal, 1999; Thelan, 1992). This happens in areas as diverse as number conservation (Goldin-Meadow, Alibali, & Church, 1993), mastering grammatical rules (Bowerman, 1982), vocabulary expansion (Gershkoff-Stowe, & Smith, 1997) and word combination (Winjin, 1990). For example, dramatic vocabulary growth is seen during the so-called "word spurt" which occurs when toddlers have a spoken or productive vocabulary of 50 to 100 words, or around 18 months of age (Bates, Bretherton, & Snyder, 1988). Naming errors - incorrect labels for familiar objects that were previously named correct - are higher during this period of rapid word acquisition than before or after (Gershkoff-Stowe, & Smith, 1997).

Similar increases in variability during learning are seen earlier and later. Infants acquiring postural control experience two periods of high motor variability. The first (birth to 6 months) involves exploration of all possible movement properties (e.g., velocity, force), none of which are goal-directed, all of which are necessary for brain organization. The second (at two years) occurs when the toddler learns to make task-specific postural adjustments (Hadders-Algra, 2002). Variability continues to accompany learning in adulthood, as demonstrated during novice-to-expert transitions. In radiology (Lesgold et al., 1988), greater variability in the form of increased errors precedes the appearance of advanced diagnostic ability. Lesgold et al. (1988) attribute the increase to "points of impasse, where superficial and deep methods conflict" (p. 340). Superficial here means perceptual, deep refers to cognitive decision making. The authors also make quite clear the connection between skill development in child and adult: "as control shifts from a purely perceptual to cognitive processing, performance may appear to get worse – just as locomotion may get worse as a baby whose creeping is secure and automatic begins to toddle" (pp. 337-338).

High Variability Facilitates Learning

Importantly, greater initial variability leads to greater learning (Fujimura, 2001; Goldman & Saul, 1990; Schlagmuller & Schneider, 2002). Students who use multiple procedures learn more from instructions than those who use fewer (Coyle & Bjorklund, 1997). With number conservation, children who activate multiple strategies on a single problem acquire new strategies faster than those using one strategy (Church & Goldin-Meadow, 1986). Similarly, the number of explanations given by children during a pretest predicts percentage of correct explanations during training. More early explanations lead to more correct responses later (Siegler, 1995). Asking children for explanations generates greater learning than feedback, re-reading a text, or increased time on task (Alibali, 1999; Aleven & Kordinger, 2002; Chi et al, 1994; Siegler, 2002). While these results may well reflect increased depth of processing or weakening of incorrect strategies, notice that none of the alternatives (feedback, re-reading, increased time) require that child do anything new; only explaining requires different responses, thereby increasing variability.

Being more variable in practicing a motor skill enhances later performance as well as generalization to different conditions (Goode & Magill, 1986; Shea & Morgan, 1979; for a review, see Schmidt & Bjork, 1992). For example, 6 to 10 year old children who practiced tossing shuttlecocks to four different locations performed more accurately on a set of transfer tasks that those who practiced with one location (Moxley, 1979).

Such positive effects of initial variability on subsequent learning may be due to acquisition of a wide range of interconnected, and therefore, more accessible strategies (Siegler, 1996); greater flexibility in reinstating or recombining elements of a skill repertoire (Lee & McGill, 1983); more "exhaustive" search strategies (Doane, Sohn, & Schreiber, 1999); "richer" sets of retrieval cues (Shea & Morgan, 1979); greater sensitivity to changes in condition (Joyce & Chase, 1990; Stokes et al., 2007); openness to exploration (Simon & Bjork, 2002) or persistence (see Eisenberger for related work on learned industriousness, 1992). Finally – and basically - according to both Darwinian models of learning (Holland, Holyoak, Nisbett, & Thagard, 1987; Palmer & Donahoe, 1992) and dynamic systems theory (Thelen & Smith, 1994), variability is essential for exploration and selection. The latter theory adds the premise that instability (variability) precedes change (learning). In this view, "without variability" would mean "without learning."

Sources of High Variability

Reinforcement-related Accounts

Short-term effects. Reinforcement schedules and response requirements have immediate effects on variability. Schedules that serve to decrease reward density or frequency increase variability (Tatham, Wanchisen, & Hineline, 1993). Variability also increases when a response criterion specifies doing something new (Holman, Goetz, & Baer, 1977), or doing something differently (Neuringer, 2002). The greater the difference from prior responding required, the higher the variability (Machado, 1992).

These variability levels last only as long as a schedule or requirement remains in effect. Thus, they are similar to the temporary increases in variability seen when children who can already count or spell encounter difficult addition (Sielger & Jenkins, 1989), subtraction (Siegler & Shrager, 1984), or spelling (Kerkman & Sielger, 1993) problems. More difficult problems increased the use of back-up strategies (e.g., alternatives to retrieving a correct answer from memory). They also facilitated the discovery of more advanced strategies. For example, when children who are already counting are given "challenge" problems to solve, variability – measured as number of strategies used - increases; once a challenge is met, variability declines to its earlier level (Sielger & Jenkins, 1989).

In short, encountered after a basic skill is acquired, more difficult requirements or schedules of reinforcement only temporarily increase variability levels. However, as Stokes and colleagues have shown, difficulties encountered early in skill acquisition can establish and sustain high levels of variability.

Sustained effects. In this view (Stokes, 1995; 1999), learning how to do something includes learning how differently to do it. The *how* is the skill; the *how differently* is the habitual variability level, a preferred performance range within which responses differ from each other (Stokes & Balsam, 2001; Stokes & Harrison, 2002, Stokes et. al, 2007).

Constraints, which limit and direct search (Reitman, 1965), play an important role in acquiring a habitual level. Task constraints determine how differently something can be done; variability constraints specify how differently it must be done. Encountered early in skill acquisition, constraints contribute to the acquisition of successful strategies that establish and maintain habitual variability levels in a domain. The word mastery is important. Early variability levels that are associated with success are maintained; those that lead to failure are not.

Experimental evidence. A series of experiments using computer games supports the habitual variability model. In a game with a triangular maze, task constraints included response number and location (e.g., where paths through the maze must exit the maze). One variability constraint required that a current path differ from some number of immediately prior paths through the triangular maze. The number of different prior paths is called a lag. For example, with a lag2 constraint, a path only earned points if it differed from the last 2 paths.[1]

In all our studies, more difficult early constraints, which required trying many different things to succeed, led to higher sustained variability levels late in the games when all constraints are relaxed. For example, Stokes et al. (2007) trained one group of college students with an easy series of location-only task constraints. Another group was exposed to a difficult series that alternated location-only with combined location-lag constraints (e.g., exit at specified locations by paths that differed from 1 to 5 prior paths). At the end of the game, when the constraints were relaxed, the easier location-only group were far less variable than those the more difficult alternating group.

Other researchers have shown sustained effects of rewarding variability in school settings (Eisenberger & Armeli, 1998; Glover, 1980). For example, after initial reward for novelty in drawing or block-building ceased, children's variability in the rewarded domain remained high (Holman, Goetz, & Baer, 1977). Habitual levels can also be inferred from cognitive studies which show that after a basic skill is acquired, the locus of variability shifts, but not the level. For example, in solving addition problems, children switch to and between more efficient, but not more or fewer, strategies (Carpenter & Moser, 1982; Siegler & Jenkins, 1989).

[1] Thinking about this in terms of single-digit addition, a lag2 constraint would require solving the third in a problem series using a different strategy than used on the two prior problems. In the series 1 + 2, 2 + 3, 4 + 5, a child could retrieve the first answer, count on the second, and decompose (5 + 5 = 10 – 1 = 9) on the last. Since the lag does not require that all solutions be different, she could also retrieve on both the first and second problems.

Age-related Accounts

Plastic means moldable, flexible, variable. Plasticity at the neural level refers to how easily the brain adapts to the environment, establishing or reorganizing associative networks in more and less variable ways (Garlick, 2002). Synaptic pruning decreases plasticity; reorganization allows further adaptation to new experience (Nelson, 1999) in specific areas (Trainor, 2005; Werker & Tees, 2005). Such continued, specialized adaptation can be thought of as a kind of secondary plasticity. This kind of early variability facilitates specific kinds of learning including acquisition of the phonology and grammar of a language (Bjorkland & Green, 1992; van Geert & van Dijk, 2002) or musical pitch structure (Trainor, 2005). For example, proficiency in first (Lennenberg, 1967) and second languages (Johnson & Newport, 1989; Werker & Tees, 2005) are directly related to the age at which an individual is introduced to the language.[2] The basic armature (necessary not only for acquisition but for later development) must be constructed prior to puberty.

In other areas, including math and motor skills, younger less-skilled children often appear more variable than older more-experienced ones. The question here is: what kind of variability is involved? With experience, error variability (typical of younger children or novices) is replaced by functional or adaptive variability (typical of older children or experts). For example, Manoel and Connolly (1995, 1997) showed that as a child's grasping or timing skills mature, general action patterns (macrostructures) become stable, but individual response components (microstructures) remain variable. Such residual variability is highly functional, allowing the child to respond to changing conditions via reorganization of her skills.[3]

THE CURRENT STUDY

The reviewed reinforcement and age related studies offer non-competing, but also uncombined snapshots of variability. Thus, the primary purpose of the present study was to evaluate the relative contributions of reinforcement and age on functional and error variability in children's problem solving. To this end, we compared the performance of first, third, and fifth graders on a novel task during which variability was required and rewarded.

One prior study (Boulanger, 1990) addressed the age-variability issue in a related way. Three groups of French school children (ages 5-6, 9-10, and 14-15) earned points in a computer game for moving a cursor from the upper left to lower right corners of a square. There were 50 possible paths (task constraint). Points were earned for paths that differed from two immediately prior ones (variability constraint). Variability was calculated as number of different correct and incorrect paths: as age increased, correct paths increased and incorrect ones decreased.

These results suggested that reinforcement contingencies do not affect variability independently of age. To see if this was the case, we expanded on the earlier study in several ways. First, to evaluate the effects of age alone we included a baseline period in which variability was not required. Second, to determine how lower and higher

[2] Another consideration is the way first and second languages are learned. The first is generally by immersion; the second, by translation, which adds considerably to cognitive load.

[3] A similar pattern is seen in the musical expressivity of expert pianists: variability in their performance is based on intentional deviation from established exactitude. In contrast, variation in novice performance is unexpressive, unintentional and error-based (Sloboda, 1996).

constraints would affect age-related differences, we used three rather than one variability constraint.

Our questions involved variability, performance, and strategy. We wanted to know (1) if younger children are more or less variable than older children during baseline; (2) if functional variability is reward- or age-dependent or both; and (3) if strategy use changes with age.

METHOD

Participants

22 first grade (10 male, 12 female; mean age, 6.4 years), 30 third grade (17 male, 13 female; mean age, 8.5 years), and 22 fifth grade (14 male, 8 female; mean age, 10.7 years) students from the Washington School in Lodi, New Jersey participated. All children were comfortable with, and competent at, using computers

Materials

Three laptops (PCs) were used for playing the game. As shown in Figure 1, the display was a triangle (hereafter called the pyramid) with 32 paths from the apex to one of the six end points at the base. The end points are identified by letters (A to F). The number under each letter shows how many different paths lead to that end point.

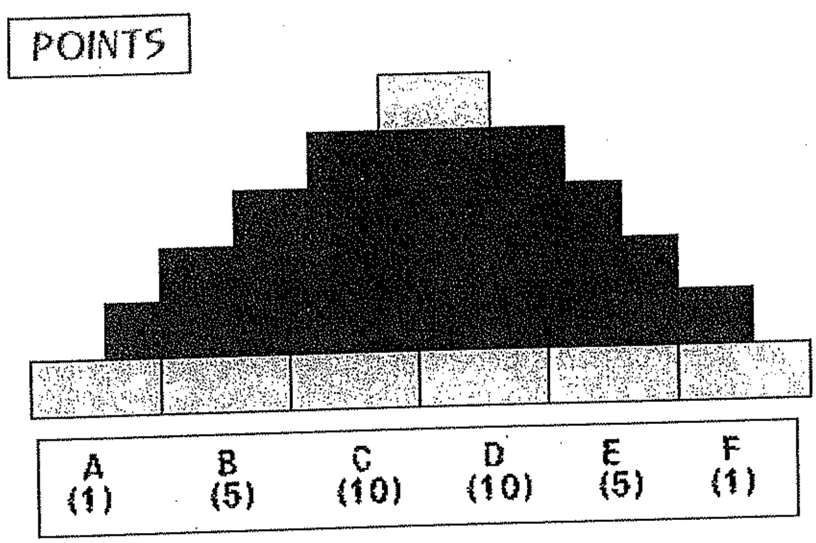

Figure 1. A Five-pyramid Display. End Points (not Included in the Display) Are Indicated by Letters A to F. Total Paths Leading to Each Are Shown in Parentheses. The Moveable Box at the Apex Was Yellow. The Middle Rows Were Black. The Bottom Row Was Red.

Pressing the left (L) or right (R) directional arrow moves the yellow box at the apex downward to the left or the right. Five presses constitute one complete path. For example, five right presses (RRRRR) produce the path leading to the F end point. Which paths are "correct" - and therefore earn points - change as the game progresses.

These requirements are discussed in the Procedure section. Whenever a correct path is taken, the words "One Point" appear beneath the score box in the upper left corner of the screen, and the total points in the box increases by one. Pressing the up arrow returns the yellow box to the apex of the pyramid. At the end of the game, the words "the game is over" appear on the screen.

Procedure

The experiment was conducted in a small classroom with moveable desks arranged so that no child could see any other's computer screen. Children came to the room in groups of three to play the game. There were three task constraints: (1) the number of responses: five presses on left or right arrow keys were required to move the cursor from the apex to the base of the pyramid; (2) the number of paths leading to each end point: these are shown in Figure 2, there were 32 in all; (3) the number of points earned for correct paths: the total was 100.

There were five variability constraints; each specified the number of prior (left-right) paths from which a current path had to differ. This number is called a lag. For example, with a 5-press task and a Lag0 constraint, any path would earn a point. With a Lag2 constraint, reward would only follow a path that differed from two immediately prior paths, for example, the third path in the sequence RRRRR, RRRRL, RRRLL.

Twenty points were earned for paths meeting each of the variability constraints shown in Table 1. These increased four times: from 0 to 2, 2 to 5, 5 to 10, and 10 to 20, making the total number of points earned 100. The total number of points was based on preliminary testing which showed that children remained focused on the game for 100 points.

Table 1
Successive Variability Constraints

Lag0	all paths were correct
Lag2:	a path had to differ from two prior paths
Lag5	a path had to differ from five prior paths
Lag10:	a path had to differ from ten prior paths
Lag20	a path had to differ from twenty prior paths

There was no time limit for earning the 100 points in the game, and all children earned 100 points. Children were tested individually by Barnard undergraduate research assistants, who introduced themselves before reading the following instructions:

"Let's pretend that all the lights in Lodi have gone out. This yellow box is you and your flashlight. You have the only flashlight in the Washington School. No one else can see where they're going, so you have to help your teachers and your friends get out.

You can only help one person at a time. The way you search is by pressing the left and the right arrow keys. You can use both keys, but you can only press one at a time and you can only use one hand. One hand, okay? And don't hold the keys down.

If you get to the bottom - here, where the red exit lights are - with someone that you found, a point will be added to your total. You get back to the top by pressing the up arrow. There are 100 people to get out of the school.

I'll just sit here while you play. The game starts when you do."

Notice three things about the instructions. First, they were specific about getting to the bottom ("where the red exit lights are"), but not in the number of presses required to do so. The number was always five. Second, the children had to discover the relationship between variability and reward for themselves. During debriefing, we learned that this inference always assumed the form of a concrete representation, i.e., go to different exits. Third, the Washington School in Lodi is the location where the children actually played the game.

Research assistants took notes on what each child said during and after the game, as well as on visually obvious strategies (for example, moving systematically from one end point to the next). At the end of the game, two specific questions were posed. The questions asked for the same information in two different ways, i.e., "Can you tell me how you found all those people?" and "Did you have a plan?"

After they played the game, children who finished before the others were given a simple drawing/coloring task. They were presented with two 5 by 5 grids on a piece of white paper and two magic markers (red and black), and simply asked to "make some designs." When all the children played the game, they returned to their classroom and another group was sent to the experimental room. Between 9 and 12 children were tested on each Friday morning that the research team visited the school.

Measures

Number of different paths is our measure of variability. Number of incorrect paths is our measure of error. Since the program did not record time, total number of paths is used to indicate how long it took to play the game.

RESULTS

Quantitative Data

Variability. Figure 2 presents mean number of different paths taken by each grade during the successive lag requirements. Two questions were asked of the data.

Did baseline levels of variability differ between grades? A one-way ANOVA with grade as the main factor was run on variability data (number of different paths) during Lag0. Grade was significant [$F(2,71) = 3.344$ $p < .05$]. Fisher's LSD test was used for post-hoc comparisons. During Lag0, when variability was not required 1st graders were less variable than 5th graders ($p < .05$). Third-graders did not differ from either 1st or 5th graders.

Did variability levels differ when the lags increased? A mixed two-way ANOVA with grade (1, 3, 5) and lag (2, 5, 10, 20) as main factors was run on variability data during the four remaining lags. There was a main effect of lag [$F(3,69) = 69.679$, $p < .01$, $n^2 = .752$]. There were no between group differences [$F(2,71) = .621$, $p = .540$, $n^2 = .017$]. All groups increased their variability as the lags increased.

Error. Figure 3 presents mean number of incorrect paths taken by each grade during the successive lag requirements. One question was asked of this data.

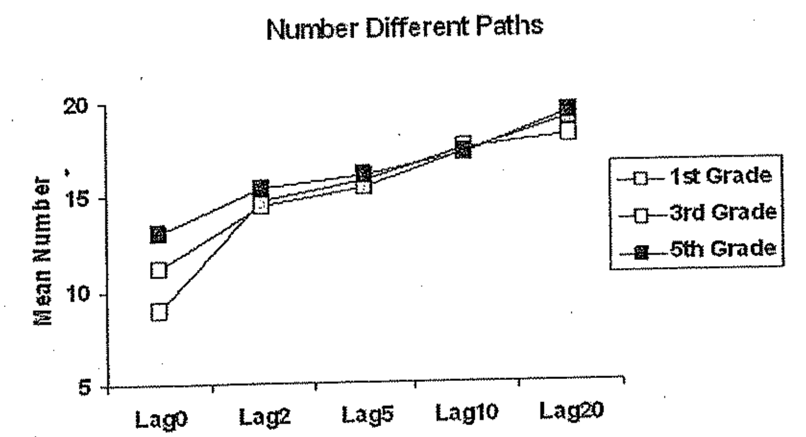

Figure 2. Mean Number of Different Paths for Each Grade during Successive Lags.

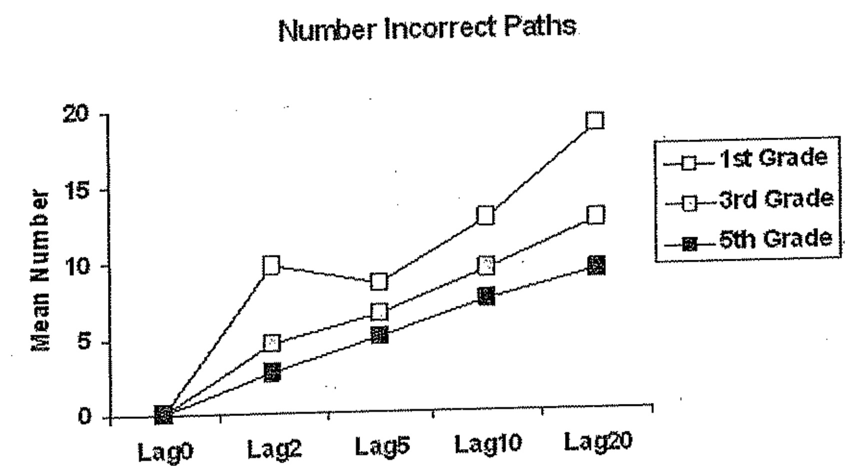

Figure 3. Mean Number of Incorrect Paths for Each Grade during Successive Lags.

Did error levels differ by grade once the lags increased? A mixed two-way ANOVA with grade (1, 3, 5) and lag (2, 5, 10, 20) as main factors was run on error data (number of incorrect paths). It produced main effects of lag [$F(3,69) = 12.946$, $p < .01$, $n^2 = .360$] and grade [$F(2,71) = 8.056$, $p < .01$, $n^2 = .185$]. In all groups, error scores increased as lags increased. Post-hoc Fisher LSD tests showed that 1st graders made more errors than either 3rd or 5th graders ($ps < .01$) who did not differ from each other. There were no significant interactions.

Total Paths/Time. Since we were unable to record the actual time each child took to complete the game, total number of paths is used as a rough equivalent. Figure 4 presents mean number of total paths taken by each grade during the successive lag requirements. One question was asked of this data.

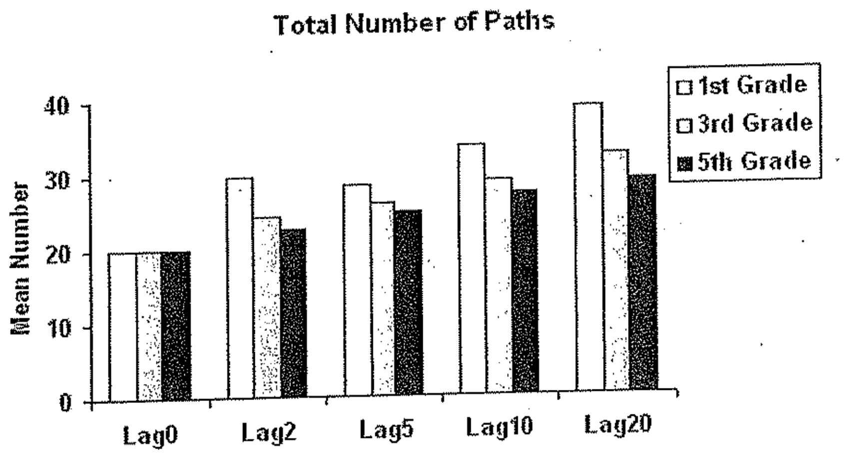

Figure 4. Mean Number of Total Paths for Each Grade during Successive Lags.

Did total number of paths differ by grade as the lags increased? A mixed two-way ANOVA with grade (1, 3, 5) and lag (2, 5, 10, 20) as main factors was run on this data (total number of paths). It produced main effects of lag [$F(3,69) = 14.355$, $p < .01$, $n^2 = .384$] and grade [$F(2,71) = 9.270$, $p < .01$, $n^2 = .207$]. In all groups, total paths increased with the lags. Post-hoc Fisher LSD tests showed that 1st graders took more paths than either 3rd or 5th graders ($ps < .01$) who did not differ from each other. There were no significant interactions.

Individual Comparisons. Figure 5 shows response patterns for the best performing student in each grade. Notice the similarity in the patterns: number of different paths is always higher than number of incorrect paths, all three students begin with and maintain high variability levels.

In the worst performer from each grade, shown in Figure 6, variability (# different paths) begins and remains low at all lags; errors increase as the lags increase, number of incorrect paths exceeds number of different ones at lags 10 and 20. Response patterns for the worst scorers reflected the ways in which they repeated themselves: first, reproducing a just rewarded path; second, developing a dominant path, defined as one occurring more than 20 times.

The most common dominant paths among low performers were all left (LLLLL) or all right (RRRRR). In contrast, the best scorers did not repeat just-rewarded paths, and did not develop dominant paths; the most frequent path in this group was taken only 8 times.

Group differences reflect the proportion of students whose response patterns approached the best or worst in their grade. For example, the percentages of students with error scores of 5 or less (the high performers) were: 45% in 5th grade, 33% in 3rd grade, and 9% in 1st grade. The percentages of students with higher error than variabi-

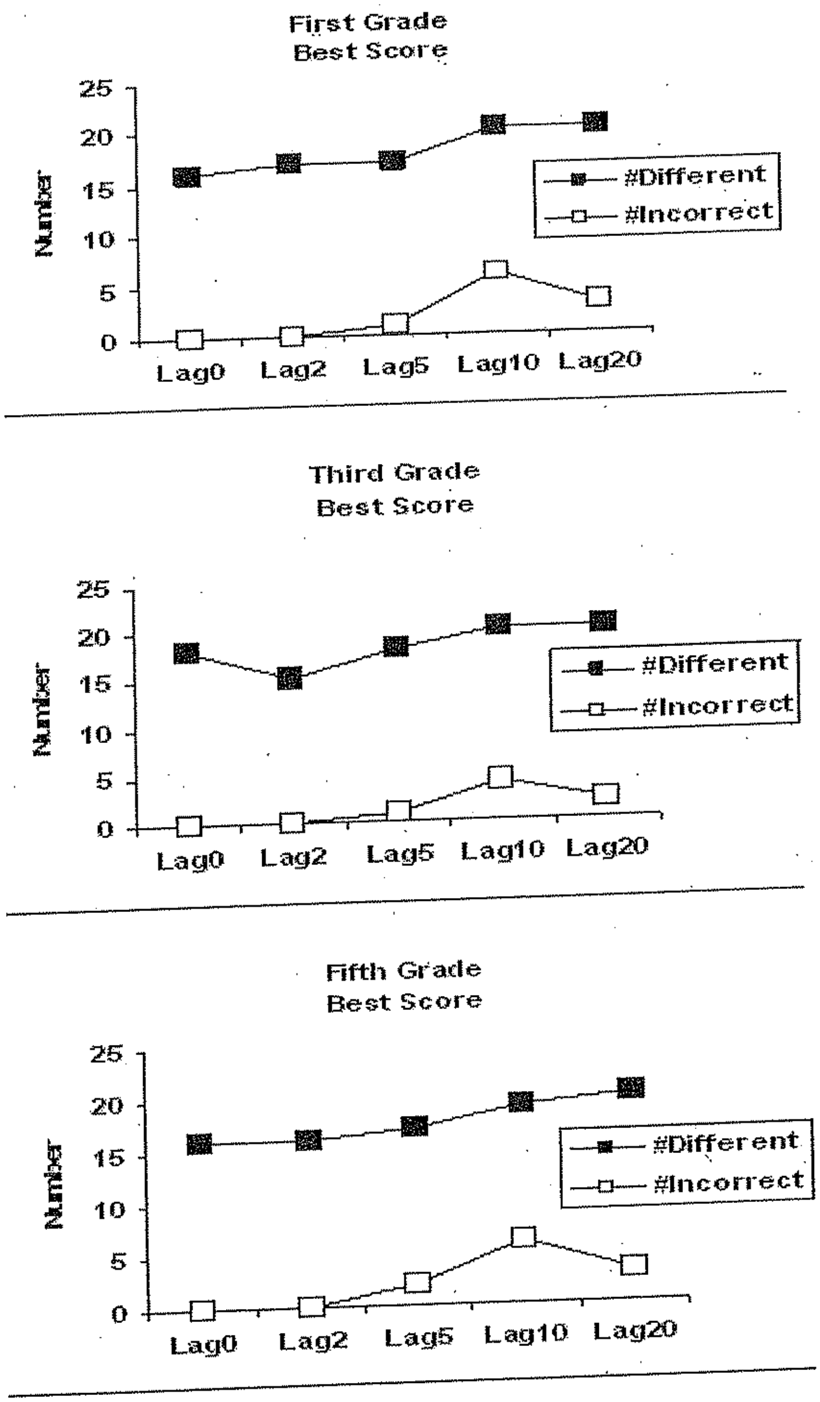

Figure 5. Best Performing Student in Each Grade.

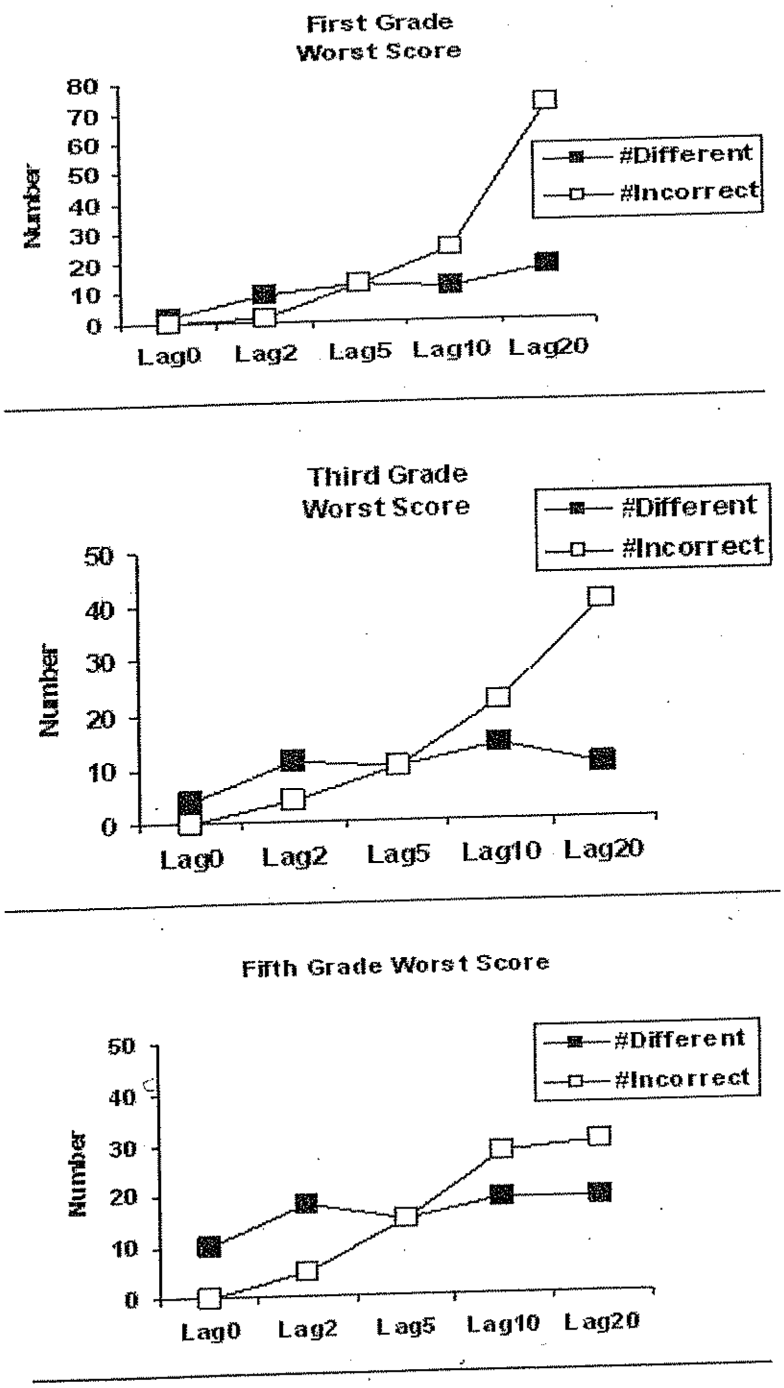

Figure 6. Worst Performing Student in Each Grade.

lity scores were: 5% in 5th grade, 20% in 3rd grade, and 45% in 1st grade. There were no systematic differences between males and females. "Can you tell me how you found all those people?" and "Did you have a plan?" Two questions were asked of this data.

Qualitative Data
Table 2 gives percentages by grade and gender for answers given to the questions

Did what students say differ by grade or by gender? Yes. As Table 2 shows, guessing was highest among 3rd graders; pretending and no plan were highest in 1st grade. Only males pretended; only females had no plan. The most efficient strategy (different exits) was used most often by fifth graders. Efficiency here is based on the relative ease of remembering and thus avoiding the exit used on a prior trial, com-pared to remembering/avoiding a particular Left-Right path. Equal percentages of males and females used this strategy.

Table 2
Self-reported Strategies

	Grade			Gender	
Strategy	First	Third	Fifth	Male	Female
Guessed	10%	24%	6%	5%	18%
No Plan	10%	-	-	-	11%
Pretended*	20%	-	6%	4%	-
Different Exits	40%	36%	60%	43%	43%
Different Paths/Patterns	15%	36%	22%	42%	21%
Different Places**	5%	4%	6%	6%	7%

* Pretended included things like "I'm going to be Dash from *The Incredibles*. I'll get everyone out fast," "This is a job for Batman. The Joker did this," (this meant shutting the lights out), or "This is an evacuation!!!"

** Different places included the girl's bathroom, Mrs. X's room, Mr. Carafa's office, the basement, and the staircases.

Did what the best and worst scorers report differ? Yes. According to self-reports by the students whose response patterns appear in Figures 6 and 7, the best scorers in all three grades reported going to different exits ("blocks on the bottom"). The worst scorers in 1st and 5th grades said they had no plan. The third grader with the worst score reported "looking for a pattern - like one left, one right, two left, two right - that worked."

DISCUSSION

Findings
The current study used a computer maze game to compare two sources of functional and error variability in children's problem solving. It replicated Boulanger's (1990)

basic finding: reinforcement contingencies did not affect variability independently of age. It expanded on the earlier work in four ways. First, it showed that the oldest children (5th graders) were more variable than the youngest (1st graders) when variability was not required. Second, that while errors (incorrect paths) increased in all grades as the requirements increased, the youngest students consistently made more errors than both older (3rd and 5th grade) groups. Third, despite no differences in functional varaiblity between third and fifth graders, more of the older group reported using the most efficient strategy (different exits).[4] Fourth, despite the overall age effects, the best scoring students in each grade were similar in response patterns and strategy use.

Explanations

While the current reinforcement contingencies increased both functional (number of different paths) and error (number of incorrect paths) variability, their effects were clearly age-related. In the age-span of the current study (6.4 to 10.7 years), experience and maturation (particularly of the pre-frontal cortex) influence problem-solving in multiple ways. Memory span increases: older children can hold more items in working memory (Henry & Miller, 1991; Siegler, 1996). Older children also perform faster on cognitive tasks (Bjorkland & Green, 1992; Chi, 1997; Kail, 1986, 1991). They generate (Bjorkland & Harnishfeger, 1987; Kee, 1994) and execute (Baker-Ward et al., Miller et al., 1991) strategies with less effort than younger children. They also more easily inhibit off-task and inefficient responses (Williams et al., 1999). With experience, more sophisticated strategy use increases (Siegler & Jenkins, 1987).

While the above discussion can account for the differences between the younger and older students in our study, questions remain regarding the strategies reported by third- and fifth-graders, and the similarities between the best performing students in all grades.

While 60% of fifth graders reported using the highly efficient different exit strategy, only 36% of third graders did so. The remainder reported taking different paths, a more cognitively demanding task in that remembering specific paths is more difficult than remembering specific exits (36%). Third graders also guessed (24%) or went to specific imaginary "places" (4%). This mix of strategies is reminiscent of the so-called utilization deficiency (Miller, 1990; Bjorkland, Coyle, & Gautney, 1992), a transitional phase marked by inefficient strategy selection. It is different in that, despite reporting high percentages of less efficient strategies, third graders were as functionally variable as fifth-graders. An alternative account of their performance involves habitual variability levels acquired via changes in school curricula; first and second graders are generally rewarded for reliable, correct responding (2 + 2 always equals 4); in third grade, more variable performance (how many ways can the numbers 1, 2, and 3 be combined to equal 4?) could become normative and thus habitual. If this were the case, faced with a novel task, a third grader's default response would be a highly variable one. If this is the case, reinforcement is an important contributor to functional variability independent of age.

Similarities among high-performing students across grade/age suggest that the current task could serve as an early indicator of giftedness, which is marked by high functional variability and focus. At minimum, it might well predict performance in

[4] While 40% of first graders also said they went to different exits, research assistants noted that several used the phrase to mean specific "doors" leading out of the "school," not to switching between exits.

visual-spatial tasks requiring focused attention in order to discern problem structures and develop appropriate strategies.

Applications

The present study demonstrates that functional variability in children's problem solving increases with age, and - quite possibly - with curricula that reinforce habitually high levels of variability. Given that high variability facilitates learning (Siegler, 1995) and transfer (Stokes et. al, 2007), and that habitual levels are established early in skill acquisition (Stokes & Balsam, 2001), all learning should take place in a context where high functional variability is the norm rather than the exception. This does not mean sacrificing reliability. For example, we have recently developed a training procedure (Stokes et al., 2007) that alternately reinforces reliability during skill acquisition with variability in application, and importantly, facilitates transfer to novel problems.

Thus, we propose that younger children (like our first graders) not only be taught a variety of strategies (for example, in math, count-all, count-on, min, etc.) but also be instructed to, and rewarded for, alternating between them.[5] Those who do not need such instruction, whose performance on novel tasks requiring high functional variability is indistinguishable from that of successful older children, could well be gifted in the areas tapped by the tasks.

REFERENCES

Adolph, K. E. (1997). Learning in the development of infant locomotion. *Monographs of the Society for Research in Child Development, 62* (3, Serial. No. 251).

Alibali, M. W. (1999). How children change their minds: Strategy change can be gradual or abrupt. *Developmental Psychology, 35*, 127-145.

Aleven, V. A., & Koedinger, K. R. (2002). An effective metacognitive strategy: Learning by doing and explaining with a computer-based cognitive tutor. *Cognitive Science, 26,* 147-179.

Baker-Ward, L., Ornstein, P. A., & Holden, D. J. (1984). The expression of memorization in early childhood. *Journal of Experimental Child Psychology, 37*, 555-575.

Bates, E., Bretherton, I., & Snyder, L. (1988). *From first words to grammar*. Cambridge, MA: Cambridge University Press.

Bertenthal, B. (1999). Variation and selection in the development of perception and action. In G. Savelsbergh, H. van der Mass, & P. van Geert (Eds.), *Non-linear developmental processes* (Vol. 175, pp. 105-121). Amsterdam, The Netherlands:

Bjorkland, D. F., Coyle, T. R, & Gaultney, J. F. (1992). Developmental differences in the acquisition and maintenance of an organizational strategy: Evidence for the utilization deficiency hypothesis. *Journal of Experimental Child Psychology, 54*, 434-448.

Bjorklund, D. F., & Green, B. L. (1992). The adaptive nature of cognitive immaturity. *American Psychologist, 47*, 46-54.

[5] We are not suggesting that children be variable in "discovering" new strategies, rather that they vary in applying ones that they already know.

Bjorklund, D. F., & Harnishfeger, K. K. (1987). Developmental differences in the acquisition and maintenance of an organizational strategy: Evidence for the utilization deficiency hypothesis. *Journal of Experimental Child Psychology, 44*, 109-125.

Boulanger, B. (1990). *La variabilite comportementale: Une approache developpementale chez l'humain* [Behavioral variability: A developmental approach]. Unpublished doctoral dissertation. Faculte de Psychologie et de Sciences de l'Education, Universite de Liege, Liege, Belgium.

Bowerman, M. (1982). Starting to talk worse: Clues to language acquisition from children's late speech errors. In S. Strauss (Ed.), *U-shaped behavior growth* (pp. 101-145). NY: Academic Press.

Carpenter, T. P, & Moser, J. M. (1982). The development of addition and subtraction problem-solving skills. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 9-24). Hillsdale, NJ: Erlbaum.

Chi, M. T. H., de Leeuw, N., Chiu, M. H., & LaVancher, C. (1994). Eliciting self-explanations improves understanding. *Cognitive Science, 18*, 439-477.

Church, R. B., & Goldin-Meadow, S. (1986). The mismatch between gesture and speech as an index of transitional knowledge. *Cognition, 23*, 43-71.

Doane, S. M., Sohn, Y. W., & Schrieber, B. (1999). The role of processing strategies in the acquisition and transfer of a cognitive skill. *Journal of Experimental Psychology: Human Perception and Performance, 25*, 1390-1410.

Eisenberger, R. (1992). Learned industriousness. *Psychological Review, 99*, 248-267.

Eisenberger, R., & Armeli, A. (1998). Can salient reward increase creative performance without reducing intrinsic creative interest? *Journal of Personality and Social Psychology, 72*, 704-714.

Fujimura, N. (2001). Facilitating children's proportional reasoning: A model of reasoning processes and effects of intervention on strategy change. *Journal of Educational Psychology, 93*, 589-603.

Garlick, D. (2002). Understanding the nature of the general factor of intelligence: The role of individual differences in neural plasticity as an explanatory mechanism. *Psychological Review, 109*, 116-136.

Gershkoff-Stowe, L. & Smith, L. B. (1997). A curvilinear trend in naming errors as a function of early vocabulary. *Cognitive Psychology, 34*, 37-71.

Glover, J. A. (1980). A creativity training workshop: Short-term, long-term, and transfer effects. *Journal of Genetic Psychology, 136*, 3-16.

Goldin-Meadow, S., Alibali, M. W., & Church, R. B. (1993). Transitions in concept acquisition: Using the hand to read the mind. *Psychological Review, 100*, 279-297.

Goldman, S. R., & Saul, E. U. (1990). Flexibility in text processing: A strategy competition model. *Child Development, 58*, 601-622.

Goode, S., & Magill, R. A. (1986). The contextural interference effects in learning three badminton serves. *Research Quarterly for Exercise and Sport, 57*, 308-314.

Hadders-Algra, M. (2002). Variability in infant motor behavior: A hallmark of the healthy nervous system. *Infant Behavior & Development, 25*, 433-451.

Henry, L. A., & Miller, S. (1991). Memory span increases with age: A test of two hypotheses. *Journal of Experimental Child Psychology, 51*, 459-484.

Hermelin, B., & O'Connor, N. (1986). Spatial representations in mathematically and in artistically gifted children. *British Journal of Psychology, 56*, 150-157.

Holland, J. H., Holyoak, K. J., Nisbett, R. E., & Thagard, P. R. (1987). *Induction: Processes of inference, learning, and discovery*. Cambridge, MA: MIT Press.

Holman, J., Goetz, E. M., & Baer, D. M. (1977). The training of creativity as an operant and an examination of its generalization characteristics. In B.C. Etzel, J. M. LeBlanc & D.M. Baer (Eds.), *New developments in behavioral research: Theory, methods and applications* (pp. 441-471). Hillsdale, NJ: Erlbaum.

Johnson, J. S., & Newport, E. L. (1989). Critical period effects in second language learning: The influence of motivational style on the acquisition of English as a second language. *Cognitive Psychology, 21*, 60-99.

Joyce, J. H., & Chase, P. N. (1990). Effects of response variability on the sensitivity of rule-governed behavior. *Journal of the Experimental Analysis of Behavior, 54*, 251-262.

Kail, R. (1986). Sources of age differences in speed of processing. *Child Development, 57*, 969-987.

Kail, R. (1991). Processing time declines exponentially during childhood and adolescence. *Developmental Psychology, 27*, 259-266.

Kee, D. W. (1994). Developmental differences in associative memory: Strategy use, mental effort, and knowledge-access interactions. In H.E. Reese (Ed.), *Advances in child development and behavior* (Vol. 25, pp. 7-32). NY: Academic Press.

Kerkman, D. D., & Siegler, R. S. (1993). Individual differences and adaptive flexibility in lower-income children's strategy choices. *Learning and Individual Differences, 5*, 113-136.

Lee. T. D., & Magill, R. A. (1983). The locus of contextural interference in motor-skill acquisition. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 9*, 730-746.

Lennenberg, E. H. (1967). *Biological foundations of language*. NY: Wiley.

Lesgold, A., Rubinson, H., Feltovich, P., Glaser, R., Klopfer, D., & Wang, Y. (1988). Expertise in a complex skill: Diagnosing x-ray pictures. In M.T.H. Chi, R. Glaser, & M.J. Farr (Eds.), *The nature of expertise* (pp. 311-342). Hillsdale, NJ: Erlbaum.

Manoel, E.deJ., & Connolly, K.J. (1997). Variability and stability in the development of skilled actions. In K.J. Connolly & H. Forssberg (Eds.), *Neurophysiology and neuropsychology of motor development* (pp. 286-318). London: MacKeith Press.

Manoel, E.deJ., & Connolly, K. J. (1995). Variability and the development of skilled actions. *International Journal of Psychophysiology, 19*, 129-147.

Machado, A. (1992). Behavioral variability and frequency-dependent selection. *Journal of the Experimental Analysis of Behavior, 58*, 241-263.

Milbrath, C. (1998). Patterns of artistic development in children: Comparative studies of talent. Cambridge, UK: Cambridge University Press

Miller, P. H. (1990). The development of strategies of selective attention. In D.F. Bjorkland (Ed.), *Children's strategies:Contemporary views of cognitive development* (pp. 157-184). Hillsdale, NJ: Erlbaum.

Miller, P. H. & Aloise-Young, P. A. (1995). Preschoolers' strategic behavior and performance on a same-different task. *Journal of Experimental Child Psychology, 60*, 284-303.

Miller, P. H., Woody-Ramsey, J., & Aloise, P. A. (1991). The role of strategy effortfulness in strategy effectiveness. *Developmental Psychology, 27*, 738-745.
Moxley, S. E. (1979). Schema: The variability of practice hypothesis. *Journal of Motor Behavior, 11*, 65-70.
Nelson, C. A. (1999). Neural plasticity and human development. *Current Directions in Psychological Science, 8*, 42-45.
Neuringer, A. (2002). Operant variability: Evidence, functions, and theory. *Psychonomic Bulletin and Review, 9*, 672-705.
Palmer, D. C., & Donahoe, J. W. (1992). Essentialism and selectionism in cognitive science and behavior analysis. *American Psychologist, 47*, 1344-1358.
Piaget, J. (1955). The stages of intellectual development in childhood and adolescence, (Reprinted in The essential Piaget, by H. E. Gruber & J. J. Voneche, Eds., 1977, pp. 814-819, London: Routledge and Kegal Paul).
Rittle-Johnson, B. & Siegler, R. S. (1999). Learning to spell: variability, choice, and change in children's strategy use. *Child Development*, 70, 332-348.
Reitman, W. R. (1965). *Cognition and thought: An information processing approach.* NY: Wiley.
Schlagmuller, M., & Schneider, W. (2002). The development of organizational strategies in children: Evidence from a microgenetic longitudinal study. *Journal of Experimental Child Psychology, 81*, 298-319.
Schauble, L. (1996). The development of scientific reasoning in knowledge-rich contexts. *Developmental Psychology, 32*, 102-119.
Schmidt, R. A., & Bjork, R. A. (1992). New conceptualizations of practice: Common principles in three paradigms suggest new concepts for training. *Psychological Science, 3*, 207-217.
Shea, J. B., & Morgan, R. L. (1979). Contextual interference effects on the acquisition, retention, and transfer of a motor skill. *Journal of Experimental Psychology: Human Learning and Memory, 5*, 179-187.
Siegler, R. S. (1994). Cognitive variability: A key to understanding cognitive development. *Current Directions in Psychological Science, 3*, 1-5.
Siegler, R. S. (1995). How does change occur? A microgenetic study of number conservation. *Cognitive Psychology, 28*, 225-273.
Siegler, R. S. (1996). *Emerging Minds: The process of change in children's thinking.* NY: Oxford U., Press
Siegler, R. S. (2002). Microgenetic studies of self-explanation. In N. Granott & J. Parziale (Eds.), *Microdevelopment:Transition processes in development and learning* (pp. 31-58). NY: Cambridge University Press.
Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*. Hillsdale, NJ: Erlbaum.
Siegler, R. S., & Schrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do. In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229-293). Hillsdale, NJ: Erlbaum.
Simon, D. A., & Bjork, R. A. (2002). Models of performance in learning multisegment movement tasks: Consequences for acquisition, retention, and judgments of learning. *Journal of Experimental Psychology: Applied, 8*, 222-232.
Sloboda, J. A. (1996). The acquisition of musical performance expertise: Deconstructing the "talent" account of individual differences in musical expressivity. In K. A. Erricsson (Ed.), *The road to excellence: The acquisition of expert per-*

formance in the arts and sciences, sports and games (pp.107-126). Mahwah, NJ: Erlbaum.

Stokes, P. D. (1995). Learned variability. *Animal Learning and Behavior, 23*, 164-176.

Stokes, P. D. (1999). Learned variability levels: Implications for creativity. *Creativity Research Journal, 12*, 37-45.

Stokes, P. D. (2001). Variability, constraints, and creativity: Shedding light on Claude Monet. *American Psychologist, 36*, 355-359.

Stokes, P. D. (2005). *Creativity from constraints: The psychology of breakthrough.* NY: Springer.

Stokes, P. D., & Balsam, P. (2001). An optimal period for setting sustained variability levels. *Psychonomic Bulletin and Review, 8*, 177-184.

Stokes, P. D., & Harrison, H. (2002). Constraints have different concurrent and after-effects on variability. *Journal of Experimental Psychology: General, 131*, 552-566.

Stokes, P. D., Lai, B., Holtz, D. Rigsbee, E., & Cherrick, D. (2007). Effects of practice on variability. Effects of variability on transfer. *Journal of Experimental Psychology: Human Perception and Performance* (in press).

Tatham, T. A., Wanchisen, B. S., & Hineline, P. N. (1993). Effects of fixed and variable ratios on human variability. *Journal of the Experimental Analysis of Behavior, 59*, 349-359.

Thelan, E. (1992). Development as a dynamic system. *Current Directions in Psychological Science, 1*, 189-193.

Thelan, E., & Smith, L. B. (1994). *A dynamic systems approach to the development of cognition and action.* Cambridge, MA: MIT Press.

Trainor, L. J. (2005). Are there critical periods for musical development? *Developmental Psychobiology, 46*, 262-278.

van Geert, P., & van Dijk, M. (2002). Focus on variability: New tools to study intra-individual variability in developmental data. *Infant Behavior and Development, 25*, 340-374.

Vygotsky, L. S. (1987). *The collected works of L.S. Vygotsky.* NY: Plenum.

Werker, J. F., & Tees, R. C. (2005). Speech perception as a window for understanding plasticity and commitment in language systems of the brain. *Developmental Psychobiology, 46*, 233-251.

Williams, B. R., Ponesse, J. S., Schachar, R. J., Logan, G. D., & Tannock, R. (1999). Development of inhibitory control over the life span. *Developmental Psychology, 35*, 205-213.

Winjin, F. (1990). The development of sentence planning. *Journal of Child Language, 17*, 651-675.

Winner, E. (1996). *Gifted children: Myths and realities.* NY: Basic Books.

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